k-Closest Coverage Probability and Area Spectral Efficiency in Clustered D2D Networks

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***Abstract*—In this paper, we develop a comprehensive analytical framework to characterize the performance of device-centric content availability in device-to-device (D2D) networks. Modeling the locations of devices as a variant of Thomas cluster process, we derive the coverage probability of a typical device when its content of interest is available at its k^{th} closest device within the same cluster. Using the coverage probability results, we characterize the area spectral efficiency (ASE) of the whole network. A key intermediate step in this analysis is the derivation of the distributions of distances from the typical device to both the intra- and inter-cluster devices. Our analysis reveals that an optimum number of D2D transmitters must be simultaneously activated per cluster in order to maximize ASE. This can be interpreted as the classical tradeoff between more aggressive frequency reuse and higher interference power. Our analysis also quantifies the best and worst case performance of the clustered D2D networks both in terms of coverage and ASE.**

***Index Terms*—Device-to-device (D2D) communication, clustered D2D network, Thomas cluster process, stochastic geometry.**

I. Introduction

D2D communication enables direct communication between proximate devices, thereby enhancing cellular networks performance by improving overall spectrum utilization and traffic offloading [1]. *Content centric* nature of D2D communication opens up several exciting possibilities that were not quite possible with traditional cellular architecture. This is primarily driven by the spatiotemporal correlation in the content demand [2]. In particular, when a device downloads a *popular* file, it can deliver it locally to its proximate devices whenever they need it [3]–[5]. We term each such set of proximate devices as a *cluster*. The performance of a typical D2D link within each cluster will mainly depend upon where the content of interest is available with respect to the typical receiver and the interference due to other active D2D links in the network. The main goal of this paper is to develop comprehensive framework for the modeling and analysis of a device-centric content availability setup where the content of interest for a typical device is located at its k^{th} closest device. This is henceforth referred to as *k-closest content availability*.

Motivation and Related Work. Modeling and analysis of D2D communication has taken two main directions in the literature. The first one focuses on characterizing the scaling of per-device throughput as a function of the network size; see [4]–[6] for a small subset. To maintain analytical tractability, *protocol model* is typically assumed under which the transmission between two devices is successful only if the distance between them is smaller than a certain predefined value. The second direction, which is also more relevant to our work, focuses on characterizing metrics, such as the distribution of per-device throughput and coverage probability, using tools from stochastic geometry under more general physical layer models in which the metrics are defined in terms of the actual received powers from the desired and interfering devices, as opposed to Euclidean distances that appear in the protocol model [7]–[10]. The common approach in all these works is to model the locations of the the D2D transmitters (Txs) as a Poisson Point Process (PPP) while two approaches are considered for modeling the locations of the D2D receivers (Rxs). In the first approach, to lend analytical tractability, the D2D-Rxs are located at a fixed distance from the D2D-Txs [8], [9]. Although this is a good first order model, the assumption of fixed link distance is quite restrictive. This assumption is relaxed by assuming that the intended D2D-Rx is uniformly distributed within a circle around its serving D2D-Tx [7], [10]. However, neither of these stochastic geometry-based approaches captures the possibility of having multiple proximate devices, i.e., *cluster*, any of which can act as a serving device for a given device, which is quite fundamental to D2D networks [4]–[6]. Recently, we addressed these shortcomings by developing a new and more realistic spatial model for D2D networks in which the locations of devices are modeled as a Poisson clusters process where the content of interest for the *typical* device is available inside the cluster uniformly at random [11]. Extending our work, we generalized the setup to *k-closest content availability*, where the content of interest for typical device is available at its k^{th} closest device inside the same cluster. More details of the main contributions are given next.

Contributions and Outcomes. We derive an exact expression and several approximations for coverage probability of a typical device and ASE for the *k-closest content availability* setup. As intermediate results, we characterize the new distance distributions from a typical device to its serving device and intra- and inter-cluster interfering devices that is key enabler of our analysis. Our analysis leads to several system design guidelines. First, it reveals the existence of the optimal number of links that must be activated per cluster in order to maximize ASE. This can be interpreted as the classical tradeoff between more aggressive frequency reuse and higher interference power. For typical operational regimes of interest for D2D networks, our results reveal that significant gains can

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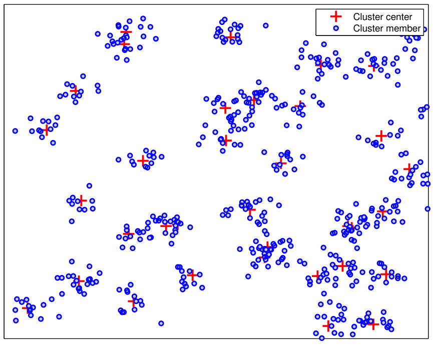

Fig. 1: Proposed D2D cluster model where devices are normally distributed around each cluster center.

be achieved by activating optimum number of links compared to strictly orthogonal strategy in which only one link per cluster is active. By tuning the value of k, we also characterize the best and worst case performances of the clustered D2D network in terms of coverage probability and ASE.

II. System Model

The locations of the devices are modeled by a *Poisson cluster process* where cluster center process is modeled by a homogeneous PPP $\Phi_{\rm c}$ with density $\lambda_{\rm c}$, and the cluster member processes (one per center) are conditionally independent [12]. In particular, the cluster members are assumed to be independent and identically distributed (i.i.d.) according to a symmetric normal distribution with variance σ^2 around each cluster center. Therefore, density function of device location relative to its cluster center, y, is

$$f_Y(y) = \frac{1}{2\pi\sigma^2}\exp\left(-\frac{\|y\|^2}{2\sigma^2}\right) \quad y \in \mathbb{R}^2. \tag{1}$$

This model is illustrated in Fig. 1. If the number of devices in each cluster were Poisson distributed, this process is simply a *Thomas cluster process* [13]. However, to simplify certain *order statistics* arguments in the sequel, we assume that the total number of devices per cluster is fixed and equal to N.

In this setup, the set of all devices in a cluster $x \in \Phi_{\rm c}$, denoted by $\mathcal{N}^x$, is partitioned randomly into two subsets: (i) set of possible *transmitting devices* denoted by $\mathcal{N}_{\rm t}^x$, and (ii) set of possible *receiving devices* denoted by $\mathcal{N}_{\rm r}^x$. The set of simultaneously transmitting devices in this cluster is denoted by $\mathcal{B}^x \subseteq \mathcal{N}_{\rm t}^x$, where $|\mathcal{B}^x|$ is assumed to be Poisson distributed with mean $\bar{m}$ conditioned on $|\mathcal{B}^x| \leq |\mathcal{N}_{\rm t}^x|$. Note that in the limiting case, about half of the devices in each cluster will transmit to the other half. Therefore, we assume that the total number of transmitting devices per cluster is limited to $M = N/2$. Without loss of generality, we perform analysis for a *typical device*, which is a randomly chosen device in a randomly chosen cluster, termed *representative cluster* centered at $x_0 \in \Phi_{\rm c}$. For notational simplicity, we assume that the typical device is located at the origin where the content of interest to the typical device is available at its k^{th} closest device from the set $\mathcal{N}_{\rm t}^{x_0}$ in the same cluster. By tuning the value of k in this generalized content availability setup, termed as *k-closest content availability*, the content can be biased to lie closer (small k) or farther (large k) from the typical device that covers all the possible position of serving device. After fixing the location of the serving device, the intra-cluster interfering devices are sampled uniformly at random from the remaining $M-1$ devices in $\mathcal{N}_{\rm t}^{x_0}$ in the representative cluster. Since a representative cluster has a serving device by definition, for concreteness we assume that the number of interfering devices is Poisson distributed with mean $\bar{m}-1$. Similarly, the inter-cluster interfering devices are sampled uniformly at random from the set of transmitting devices of each cluster, such that the number of active devices in each cluster is Poisson distributed with mean $\bar{m}$ conditioned on the total number of transmitting devices being less than M.

Now, assume that serving device is located at distance $r = \|y_0 + x_0\|$ from typical device and each device transmit with power $P_{\rm d}$, the received power at a typical device is

$$P = P_{\rm d} h_0 \|x_0 + y_0\|^{-\alpha}, \tag{2}$$

where $h_0 \sim \exp(1)$ is i.i.d. exponential random variable which models Rayleigh fading and α is path loss exponent. To define interference field, it is useful to define the set of all simultaneously active D2D-Txs as:

$$\Psi_{\rm m} = \cup_{x\in\Phi_{\rm c}} \mathcal{B}^x, \tag{3}$$

where recall that $\mathcal{B}^x$ is the set of simultaneously active transmitting device inside a cluster $x \in \Phi_{\rm c}$. In this network, the total interference caused at the typical device can be written as the sum of two independent terms: (i) intra-cluster interference caused by the interfering devices inside the representative cluster, and (ii) inter-cluster interference caused by simultaneously active transmitting devices outside the representative cluster. The intra-cluster interference power can be expressed as:

$$I_{\rm Tx-cluster} = \sum_{y\in\mathcal{B}^{x_0}\setminus y_0} P_{\rm d} h_{y_{x_0}} \|x_0 + y\|^{-\alpha}. \tag{4}$$

Similarly, inter-cluster interference power can be expressed as:

$$I_{\Psi_{\rm m}\setminus{\rm Tx-cluster}} = \sum_{x\in\Phi_{\rm c}\setminus x_0}\sum_{y\in\mathcal{B}^x} P_{\rm d} h_{y_x}\|x + y\|^{-\alpha}. \tag{5}$$

Hence, the signal-to-interference ratio (SIR) experienced by the typical device is

$$\texttt{SIR}(r) = \frac{P_{\rm d} h_0 r^{-\alpha}}{I_{\Psi_{\rm m}\setminus{\rm Tx-cluster}} + I_{\rm Tx-cluster}}. \tag{6}$$

For notational simplicity, we assume that the background noise is negligible compared to the interference and is hence ignored. This means that the transmit power term cancels in the SIR expression and can hence be set $P_{\rm d} = 1$.

III. k-closest Content Availability Analysis

This is the main technical section of the paper where we first characterize the distributions of the distances from the typical device to various intra- and inter-cluster devices. These distance distributions will be used in the analysis of coverage probability and ASE later in this section.

A. Distribution of the Distances

Before going into more technical details, we first define the functional forms of the probability density functions (PDFs) of the Rayleigh and Rician distributed random variables, which will significantly simplify the notation in the rest of the paper.

Definition 1 (Rayleigh distribution)**.** *The PDF of the Rayleigh distributed random variable is*

$$\texttt{Raypdf}(a;\sigma^2) = \frac{a}{\sigma^2}\exp\left(-\frac{a^2}{2\sigma^2}\right), \quad a>0, \tag{7}$$

where σ is the scale parameter of the distribution.

Definition 2 (Rician distribution)**.** *The PDF of the Rician distributed random variable is*

$$\texttt{Ricepdf}(a,b;\sigma^2) = \frac{a}{\sigma^2}\exp\left(-\frac{a^2+b^2}{2\sigma^2}\right)I_0\left(\frac{ab}{\sigma^2}\right), \quad a>0, \tag{8}$$

where $I_0(.)$ is the modified Bessel function of the first kind with order zero and σ is the scale parameter.

Now, Let's start our discussion with the intra-cluster distances by focusing on the representative cluster. Denote by $\mathcal{S}_{\rm t}^{x_0}$, the set $\{S_i\}_{i=1:M}$ of distances from the typical device to the set of possible transmitting devices $\mathcal{N}_{\rm t}^{x_0}$ in the cluster $x_0 \in \Phi_{\rm c}$, where $s_i = \|x_0+y\|$ is the realization of S_i. The ordering in this case is arbitrary, which means $S_i \in \mathcal{S}_{\rm t}^{x_0}$ will be interpreted as the distance from the typical device to a device chosen uniformly at random from $\mathcal{N}_{\rm t}^{x_0}$. Whenever this interpretation is clear, we will drop index i from s_i and S_i. Characterizing the marginal distribution of S is quite straightforward. Since x_0 and y are i.i.d. Gaussian random variables with variance σ^2, x_0+y is also Gaussian with variance $2\sigma^2$. Therefore, S is Rayleigh distributed with PDF $f_S(s) = \texttt{Raypdf}(s;2\sigma^2)$. However, this does not completely characterize $\mathcal{S}_{\rm t}^{x_0}$ because it does not capture the fact that the distances from intra-cluster devices to the typical device $\{\|x_0+y\|\}$ are correlated due to the common factor x_0. That being said, if we condition on the location of the cluster center x_0 relative to the typical device, the distances in the set $\mathcal{S}_{\rm t}^{x_0}$ are i.i.d. since the device locations are i.i.d. around the cluster center by assumption. This conditional distribution is characterized in the following Lemma. In the extended draft of this paper [14], we show that instead of conditioning on the location x_0, a "weaker" conditioning on the distance $\nu_0 = \|x_0\|$, suffices. Therefore, the statement of the Lemma is presented in terms of ν_0.

Lemma 1. *Distribution of i.i.d. sequence $\mathcal{S}_{\rm t}^{x_0}$ conditioned on the distance $\nu_0 = \|x_0\|$, the PDF of an element S chosen uniformly at random from the i.i.d. sequence $\mathcal{S}_{\rm t}^{x_0}$ is*

$$f_S(s|x_0) = f_S(s|\nu_0) = \texttt{Ricepdf}(s,\nu_0;\sigma). \tag{9}$$

Proof. Due to lack of space, the proof is delegated to extended version of this paper [14]. ■

Now using the fact that $\mathcal{S}_{\rm t}^{x_0}$ is i.i.d. sequence with sampling distribution $f_S(s|\nu_0)$, the PDF of serving distance, i.e., k^{th} closest device to the typical device follows by order statistics (see [15, eq(3)]) is formally stated below.

Lemma 2 (Serving distance)**.** *The conditional distribution of serving distance given $\nu_0 = \|x_0\|$, is*

$$f_R(r|\nu_0) = \frac{M!}{(k-1)!(M-k)!}F_S(r|\nu_0)^{k-1}f_S(r|\nu_0) \times(1-F_S(r|\nu_0))^{M-k} \tag{10}$$

with $f_S(r|\nu_0) = \texttt{Ricepdf}(r,\nu_0;\sigma)$, and $F_S(r|\nu_0) = 1 - Q_1(\frac{\nu_0}{\sigma},\frac{r}{\sigma})$, where $Q_1(a,b)$ is the Marcum Q-function.

Note that the k^{th} closest device is fixed *a priori* as the serving device and hence cannot act as an interferer. To address this issue, we divide the set of simultaneously active devices into three subsets, $\mathcal{B}^{x_0} \equiv \{\mathcal{B}_{\rm in}^{x_0}, y_0, \mathcal{B}_{\rm out}^{x_0}\}$, where the serving device is located at a distance $s_{(k)} = \|x_0+y_0\|$ from the typical device, and $\mathcal{B}_{\rm in}^{x_0}$ ($\mathcal{B}_{\rm out}^{x_0}$) denote the set of devices that are closer (farther) to the typical device compared to the serving device. We show that the distances from the typical device to the devices in $\mathcal{B}_{\rm in}^{x_0}$ ($\mathcal{B}_{\rm out}^{x_0}$) are conditionally i.i.d. and their distribution is characterized in the lemma below. This i.i.d. property will play a major role in the exact analysis of Laplace transform of intra-cluster interference in the sequel.

Lemma 3 (Intra-cluster interferer distance)**.** *For the k-closest content availability strategy,*

a) the distances from the devices in the set $\mathcal{B}_{\rm in}^{x_0}$ to the typical device, i.e., $\{w_{\rm in} = \|x_0+y\|\}$, are conditionally i.i.d., conditioned on the serving distance r and the distance $\nu_0 = \|x_0\|$ between the cluster center and the typical device, with each distance following the PDF

$$f_{W_{\rm in}}(w_{\rm in}|\nu_0,r) = \begin{cases} \frac{f_S(w_{\rm in}|\nu_0)}{F_S(r|\nu_0)}, & w_{\rm in} < r \\ 0, & w_{\rm in} \geq r \end{cases}, \tag{11}$$

where $f_S(w_{\rm in}|\nu_0) = \texttt{Ricepdf}(w_{\rm in},\nu_0;\sigma)$, and $F_S(r|\nu_0) = 1 - Q_1(\frac{\nu_0}{\sigma},\frac{r}{\sigma})$, and

b) the distances from the devices in the set $\mathcal{B}_{\rm out}^{x_0}$ to the typical device, i.e., $\{w_{\rm out} = \|x_0+y\|\}$, are conditionally i.i.d., conditioned on the serving distance r and the distance $\nu_0 = \|x_0\|$ between the cluster center and the typical device, with each distance following the PDF

$$f_{W_{\rm out}}(w_{\rm out}|\nu_0,r) = \begin{cases} \frac{f_S(w_{\rm out}|\nu_0)}{1-F_S(r|\nu_0)}, & w_{\rm out} > r \\ 0 & w_{\rm out} \leq r \end{cases}, \tag{12}$$

where $f_S(w_{\rm out}|\nu_0) = \texttt{Ricepdf}(w_{\rm out},\nu_0;\sigma)$, and $F_S(r|\nu_0) = 1 - Q_1(\frac{\nu_0}{\sigma},\frac{r}{\sigma})$.

Proof. Due to lack of space, the proof is delegated to extended version of this paper [14]. ■

We now look at the distribution of the distances from inter-cluster devices to the typical device. Recall that the inter-cluster interfering devices are chosen uniformly at random from the set of transmitting devices $\mathcal{N}_{\rm t}^{x}$ in each cluster $x \in \Phi_{\rm c}$. Denoting the distances from inter-cluster interfering devices of the cluster $x \in \Phi_{\rm c}$ to the typical device by $\mathcal{S}_{\rm t}^{x}$, it can be shown that the elements of $\mathcal{S}_{\rm t}^{x}$ are conditionally i.i.d., conditioned on the distance $\nu = \|x\|$ form the typical device to the cluster center $x \in \Phi_{\rm c}$. It follows on the same lines as Lemma 1, except that conditioning here is on $\nu = \|x\|$ and not $\nu_0 = \|x_0\|$. The result is formally stated below.

Lemma 4 (Inter-cluster interferer distance distribution)**.** *Conditioned on the distance $\nu = \|x\|$ between the cluster center $x \in \Phi_{\mathrm{c}}$ and the typical device, the distances from the inter-cluster interfering devices to the typical device $\{u = \|x + y\|, \forall y \in \mathcal{B}^x\}$ are i.i.d. with each element following the PDF given by $f_U(u|\nu) = \mathtt{Ricepdf}(u, \nu; \sigma)$.*

B. Coverage Probability and ASE *Performance*

Using the above distance distributions, we now derive the coverage probability of the typical device and the ASE of the whole network. As evident in the sequel, we need the Laplace transforms of the intra- and inter-cluster interference powers as the intermediate results for the coverage and ASE analysis.

1) Laplace Transform of Interference: We start by deriving exact expression and approximation on the Laplace transform of intra-cluster interference.

Lemma 5. *The conditional Laplace transform of the intra-cluster interference power given by* (4)*, conditioned on $\nu_0 = \|x_0\|$, is $\mathcal{L}_{I_{\mathrm{Tx-cluster}}}(s, r|\nu_0) =$*

$$\sum_{n=0}^{M-1} \sum_{l=0}^{g_{\mathrm{m}}} \binom{n}{l} p^l (1-p)^{n-l} \beta_p \mathcal{K}_{\mathrm{in}}(s, r|\nu_0)^l \mathcal{K}_{\mathrm{out}}(s, r|\nu_0)^{n-l} p_n$$

with, $\mathcal{K}_{\mathrm{out}}(s, r|\nu_0) = \int_r^\infty \frac{1}{1 + s w_{\mathrm{out}}^{-\alpha}} f_{W_{\mathrm{out}}}(w_{\mathrm{out}}|\nu_0, r) \mathrm{d}w_{\mathrm{out}},$

$$\mathcal{K}_{\mathrm{in}}(s, r|\nu_0) = \int_0^r \frac{1}{1 + s w_{\mathrm{in}}^{-\alpha}} f_{W_{\mathrm{in}}}(w_{\mathrm{in}}|\nu_0, r) \mathrm{d}w_{\mathrm{in}} \quad (13)$$

where $\beta_p = \frac{1}{I_{1-p}(n - g_{\mathrm{m}}, g_{\mathrm{m}}+1)}$, $g_{\mathrm{m}} = \min(n, k-1)$, $p = \frac{k-1}{M-1}$, $p_n = \frac{(\bar{m}-1)^n e^{-(\bar{m}-1)}}{n! \xi}$ with $\xi = \sum_{j=0}^{M-1} \frac{(\bar{m}-1)^j e^{-(\bar{m}-1)}}{j!}$, I_{1-p} is regularized incomplete beta function, $f_{W_{\mathrm{in}}}(w_{\mathrm{in}}|\nu_0, r)$ and $f_{W_{\mathrm{out}}}(w_{\mathrm{out}}|\nu_0, r)$ are given by (11) *and* (12) *respectively. Note that here zero to the zero power is defined as one.*

Proof. See Appendix A. ∎

While Lemma 5 provides an exact expression for the Laplace transform of intra-cluster interference, it is usually desirable to derive simple but tight approximation, which we do next under the following assumption.

Assumption 1 (Uncorrelated intra-cluster distances)**.** *Recall that the distances between intra-cluster devices and typical device, denoted by $\mathcal{S}_{\mathrm{t}}^{x_0}$, are correlated due to (i) the common factor x_0, (ii) the effect of excluding the k^{th} closest device (serving) from the field of possible interferers. However, the coverage analysis can be simplified significantly if these correlations are ignored, which we do as a part of this assumption. More formally, we assume that the serving and intra-cluster interferer distances are i.i.d. Rayleigh distributed with marginal distributions $f_W(w) = \mathtt{Raypdf}(w; 2\sigma^2)$.*

Under the above assumption, the Laplace transform of intra-cluster interference power is simplified next.

Corollary 1 (Approximation)**.** *Under Assumption 1, the Laplace transform of intra-cluster interference at the typical device is $\tilde{\mathcal{L}}_{I_{\mathrm{Tx-cluster}}}(s) =$*

$$\exp\left(-(\bar{m}-1) \int_0^\infty \frac{s w^{-\alpha}}{1 + s w^{-\alpha}} f_W(w) \mathrm{d}w\right), \quad (14)$$

where $f_W(w) = \mathtt{Raypdf}(w; 2\sigma^2)$.

We now state the exact result for the Laplace transform of inter-cluster interference.

Lemma 6. *The Laplace transform of inter-cluster interference power given by* (5)*, is $\mathcal{L}_{I_{\Psi_{\mathrm{m}} \setminus \mathrm{Tx-cluster}}}(s) =$*

$$\exp\Big(-2\pi\lambda_{\mathrm{c}} \int_0^\infty \Big(1 - \sum_{k=0}^{M} \rho(v)^k \frac{\bar{m}^k e^{-\bar{m}}}{k! \eta}\Big) \nu \mathrm{d}\nu\Big), \quad (15)$$

with $\eta = \sum_{j=0}^{M} \frac{\bar{m}^j e^{-\bar{m}}}{j!}$, $\rho(v) = \int_0^\infty \frac{1}{1 + s u^{-\alpha}} f_U(u|\nu) \mathrm{d}u$ where $f_U(u|\nu)$ given by Lemma 4. For $M \gg \bar{m}$, the above expression reduces to $\mathcal{L}_{I_{\Psi_{\mathrm{m}} \setminus \mathrm{Tx-cluster}}}(s) =$

$$\exp\Big(-2\pi\lambda_{\mathrm{c}} \int_0^\infty \Big(1 - \exp\Big(-\bar{m}(1 - \rho(v))\Big)\Big) \nu \mathrm{d}\nu\Big). \quad (16)$$

Proof. See Appendix B. ∎

2) Coverage Probability: The coverage probability is formally defined as the probability that SIR experienced by the typical device exceeds a certain pre-determined threshold β for successful demodulation and decoding at the receiver. It is mathematically expressed as:

$$\mathtt{P_c} = \mathbb{E}_R \left[\mathbb{P}\{\mathtt{SIR}(R) > \beta \,|\, R\}\right]. \quad (17)$$

Using the Laplace transform expressions of intra- and inter-cluster interference powers derived so far in this section, an exact expression for $\mathtt{P_c}$ is derived in the following Theorem.

Theorem 1 (Coverage probability)**.** *The coverage probability of the typical device is*

$$\mathtt{P_c} = \int_0^\infty \int_0^\infty \mathcal{L}_{I_{\psi_{\mathrm{m}} \setminus \mathrm{Tx-cluster}}}(\beta r^\alpha) \mathcal{L}_{I_{\mathrm{Tx-cluster}}}(\beta r^\alpha, r|\nu_0) f_R(r|\nu_0) f_{V_0}(\nu_0) \mathrm{d}r \mathrm{d}\nu_0, \quad (18)$$

where $f_R(r|\nu_0)$ is the PDF of serving distance given by (10)*, $f_{V_0}(\nu_0) = \mathtt{Raypdf}(\nu_0; \sigma^2)$, and the inter- and intra-cluster interference Laplace transforms are given by Lemmas 6 and 5.*

Proof. From the definition of coverage probability, we have

$$\mathtt{P_c} = \mathbb{E}_{V_0} \mathbb{E}_R \left[\mathbb{P}\left\{\frac{h_{0x_0} r^{-\alpha}}{I_{\Psi_{\mathrm{m}} \setminus \mathrm{Tx-cluster}} + I_{\mathrm{Tx-cluster}}} > \beta \,\Big|\, R, V_0\right\}\right]$$

$$\stackrel{(a)}{=} \mathbb{E}_{V_0} \mathbb{E}_R \left[\mathbb{E}\left[\exp\left(-\beta r^\alpha (I_{\Psi_{\mathrm{m}} \setminus \mathrm{Tx-cluster}} + I_{\mathrm{Tx-cluster}})\right) \Big| R, V_0\right]\right],$$

where (a) follows from $h_{0x_0} \sim \exp(1)$. The result now follows from de-conditioning over R given ν_0, followed by de-conditioning over ν_0, which is simply a Rayleigh distributed random variable due to the position being sampled from a Gaussian distribution in $\mathbb{R}^2$ around each cluster center. ∎

A simpler approximation of the coverage probability under Assumption 1 is given next.

Corollary 2 (Coverage approximation)**.** *Using the results of approximation given by* (14) *and Laplace transform of inter-cluster interference in* (16)*, the coverage probability can be approximated as* $\mathtt{P_c} \simeq$

$$\int_0^\infty \tilde{\mathcal{L}}_{I_{\mathrm{Tx-cluster}}}(\beta r^\alpha)\mathcal{L}_{I_{\psi_{\mathrm{m}}\setminus \mathrm{Tx-cluster}}}(\beta r^\alpha) f_R(r)\mathrm{d}r \qquad (19)$$

with, $f_R(r) = \frac{M!}{(k-1)!(M-k)!}F_S(r)^{k-1}(1-F_S(r))^{M-k}f_S(r)$

where $f_S(r) = \mathtt{Raypdf}(r; 2\sigma^2)$*, and* $F_S(r) = 1-\exp(-\frac{r^2}{4\sigma^2})$*.*

In the numerical results section, we will show that the approximations are remarkably tight and can in fact be treated as a proxy of the exact result if needed.

3) Area Spectral Efficiency: The $\mathtt{ASE}$ simply denotes the average number of bits transmitted per unit time per unit bandwidth per unit area. Assuming that all the D2D-Txs use Gaussian codebooks for their transmissions, we can use Shannon's capacity formula to define $\mathtt{ASE} = \lambda \log_2(1+\beta)\mathtt{P_c}$ of whole network, where λ is the density of the active transmitters and $\mathtt{P_c}$ is the coverage probability. Note that different devices may have different values of k for their serving devices depending upon the scheduling strategy. Since we are not characterizing scheduling policies explicitly, the information about k for each device is not known. Therefore, we derive $\mathtt{ASE}$ for a simpler case in which all active receivers connect to the k^{th} closest device with k being the same for all the devices. The expression leads to the several design guidelines. For instance when all devices connect to the closest (furthest) provide insights into the maximum (minimum) $\mathtt{ASE}$.

Proposition 1. *The* $\mathtt{ASE}$ *of the clustered D2D network is*

$$\mathtt{ASE} = \bar{m}\lambda_{\mathrm{c}} \log_2(1+\beta)\mathtt{P_c}, \qquad (20)$$

where $\mathtt{P_c}$ *is given by* (18) *and* $\bar{m}\lambda_{\mathrm{c}}$ *represents the average density of simultaneously active D2D-Txs inside the network.*

Remark 1 (Optimum number of simultaneously active links)**.** *Note that there is a clear tradeoff between link efficiency and cluster interference. While more active links means potentially higher* $\mathtt{ASE}$*, it also increases interference significantly.* $\mathtt{ASE}$ *can, in principle, be maximized as*

$$\mathtt{ASE}^* = \max_{\bar{m}\in 1,\ldots,M} \bar{m}\lambda_{\mathrm{c}} \log_2(1+\beta)\mathtt{P_c}. \qquad (21)$$

By solving this $\mathtt{ASE}$ *optimization problem numerically, we will demonstrate the existence of an optimal value of* $\bar{m}$ *that maximizes the* $\mathtt{ASE}$ *in the numerical results section.*

IV. Results and Discussion

As shown in Fig. 2, the analytical and simulation results match perfectly, thereby validating the accuracy of analysis. Recall that the exact expression of intra-cluster interference for the k-closest content availability strategy derived in Lemma 5 involves two summations, which complicates the numerical evaluation of the exact coverage probability expression of Theorem 1. This motivated us to approximate coverage probability in Corollary 2 . In Fig. 3, we can also observe that the approximation is fairly tight. Next, we notice that cluster size M (number of possible transmitting devices per cluster) has a conflicting effect on the performance of best and worst link cases. This is simply because of "order statistics": larger cluster size M decreases the minimum serving distance (best link) and increases the maximum serving distance (worst link). Another interesting observation can be made in Fig. 2, where the $\mathtt{ASE}$ in the best link case increases (in the range considered) with the number of simultaneously active links per cluster, thereby providing "scalability" to the D2D network.

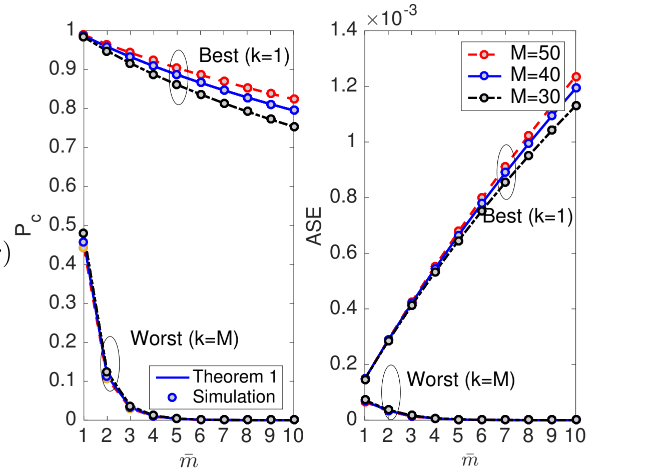

Fig. 2: Coverage probability and Analytical result of $\mathtt{ASE}$ for $\beta = 0$ dB, $\alpha = 4$, $\lambda_{\mathrm{c}} = 150$ cluster/km^2, $\sigma = 10$ and various values of transmitting cluster size M.

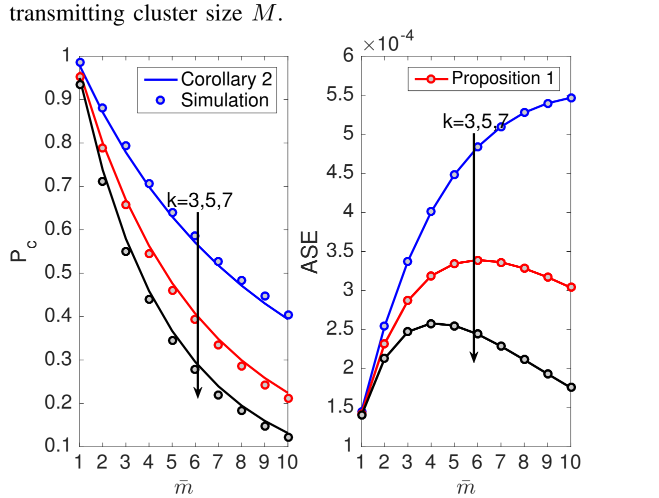

Fig. 3: Coverage probability and Analytical result of $\mathtt{ASE}$ for $\beta = 0$ dB, $\alpha = 4$, $\lambda_{\mathrm{c}} = 150$ cluster/km^2, $\sigma = 10$ and $M = 40$.

Optimum number of simultaneously active link: As shown in Fig. 3, both coverage probability and $\mathtt{ASE}$ increase significantly when the distance between the typical and serving devices is reduced, i.e., the value of k is reduced. More interestingly, we note that the optimum number of simultaneously active D2D-Txs that maximizes $\mathtt{ASE}$ also increases when the content of interest is made available closer to the typical device (smaller value of k). This highlights the importance of smarter content placement and scheduling in clustered D2D networks.

V. Conclusion

In this paper, we developed a comprehensive framework for D2D networks that is capable of capturing the fact that devices engaging in D2D may have multiple proximate devices any of which can act as a serving device. Modeling the device locations by a Poisson cluster process, we derived exact expressions and easy-to-use approximation for the coverage probability and ASE where a typical device connects to its k^{th} closest device from its own cluster. As a key intermediate step, we characterized the distributions of the distances from the typical device to various intra- and inter-cluster devices. To the best of our knowledge, this work is also the first to derive these distance distributions for the Thomas cluster process.

APPENDIX

A. Proof of Lemma 5

The Laplace transform of interference from disjoint sets of intra-cluster interferers $\mathcal{B}_{\rm in}^{x_0}$ and $\mathcal{B}_{\rm out}^{x_0}$ is $\mathcal{L}_{I_{\rm Tx-cluster}}(s,r|\nu_0) \stackrel{(a)}{=}$

$$\mathbb{E}\Big[\prod_{y\in\mathcal{B}_{\rm in}^{x_0}} \exp\left(-sh_{y_{x_0}}\|x_0+y\|^{-\alpha}\right)$$
$$\times \prod_{y\in\mathcal{B}_{\rm out}^{x_0}} \exp\left(-sh_{y_{x_0}}\|x_0+y\|^{-\alpha}\right)\Big]$$
$$\stackrel{(b)}{=} \mathbb{E}\Big[\prod_{y\in\mathcal{B}_{\rm in}^{x_0}} \frac{1}{1+s\|x_0+y\|^{-\alpha}} \prod_{y\in\mathcal{B}_{\rm out}^{x_0}} \frac{1}{1+s\|x_0+y\|^{-\alpha}}\Big]$$
$$\stackrel{(c)}{=} \mathbb{E}\Big[\underbrace{\sum_{l=0}^{g_{\rm m}} \binom{n}{l} (p)^l (1-p)^{n-l} \frac{1}{I_{1-p}(n-g_{\rm m}, g_{\rm m}+1)}}_{\mathbb{P}(l|L\le g_{\rm m})}$$
$$\Big(\underbrace{\int_0^r \frac{1}{1+sw_{\rm in}^{-\alpha}} f_{W_{\rm in}}(w_{\rm in}|\nu_0,r){\rm d}w_{\rm in}}_{\mathcal{K}_{\rm in}(s,r|\nu_0)}\Big)^l$$
$$\Big(\underbrace{\int_r^\infty \frac{1}{1+sw_{\rm out}^{-\alpha}} f_{W_{\rm out}}(w_{\rm out}|\nu_0,r){\rm d}w_{\rm out}}_{\mathcal{K}_{\rm out}(s,r|\nu_0)}\Big)^{n-l}\Big]$$
$$\stackrel{(d)}{=} \sum_{n=0}^{M-1}\sum_{l=0}^{g_{\rm m}} \mathbb{P}(l|L\le g_{\rm m})\mathcal{K}_{\rm in}(s,r|\nu_0)^l \mathcal{K}_{\rm out}(s,r|\nu_0)^{n-l} p_n,$$

with $g_{\rm m}=\min(n,k-1)$, $p=\frac{k-1}{M-1}$, $p_n=\frac{(\bar{m}-1)^n e^{-(\bar{m}-1)}}{n!\xi}$ and $\xi=\sum_{j=0}^{M-1}\frac{(\bar{m}-1)^j e^{-(\bar{m}-1)}}{j!}$, where (a) follows from the fact that $\mathcal{B}_{\rm in}^{x_0}$ and $\mathcal{B}_{\rm out}^{x_0}$ are two disjoint sets, (b) follows from expectation over $h_{y_{x_0}} \sim \exp(1)$, (c) follows from the expectation over the number of devices in $\mathcal{B}_{\rm in}^{x_0}$ and $\mathcal{B}_{\rm out}^{x_0}$, where the number of devices in $\mathcal{B}_{\rm in}^{x_0}$ is truncated binomial distribution due to the fact that $l \le \min(n,k-1)$, along with the fact that distances from interfering devices to the typical device conditioned on r and ν_0 in each set are i.i.d., and (d) from the fact that number interfering devices is Poisson distributed with mean $\bar{m}-1$ conditioned on the total being less than $M-1$.

B. Proof of Lemma 6

The Laplace transform of the aggregate interference from the inter-cluster interferers at the typical device, $\mathcal{L}_{I_{\Psi_{\rm m}\setminus{\rm Tx-cluster}}}(s)=\mathbb{E}\Big[\exp\Big(-s\sum_{x\in\Phi_{\rm c}\setminus x_0}\sum_{y\in\mathcal{B}^x} h_{y_x}\|x+y\|^{-\alpha}\Big)\Big]$ is equal to

$$= \mathbb{E}_{\Phi_{\rm c}}\Big[\prod_{x\in\Phi_{\rm c}\setminus x_0} \mathbb{E}_{\mathcal{B}^x}\Big[\prod_{y\in\mathcal{B}^x} \mathbb{E}_{h_{y_x}}\left[\exp(-sh_{y_x}\|x+y\|^{-\alpha})\right]\Big]\Big]$$
$$\stackrel{(a)}{=} \mathbb{E}_{\Phi_{\rm c}}\Big[\prod_{x\in\Phi_{\rm c}\setminus x_0} \mathbb{E}_{\mathcal{B}^x}\Big[\prod_{y\in\mathcal{B}^x} \frac{1}{1+s\|x+y\|^{-\alpha}}\Big]\Big]$$
$$\stackrel{(b)}{=} \mathbb{E}_{\Phi_{\rm c}}\Big[\prod_{x\in\Phi_{\rm c}\setminus x_0}\sum_{k=0}^{M}\Big(\int_{\mathbb{R}^2}\frac{1}{1+s\|y+x\|^{-\alpha}}f_Y(y){\rm d}y\Big)^k \times$$
$$\mathbb{P}(K=k|K<M)\Big]$$
$$\stackrel{(c)}{=} \exp\Big(-2\pi\lambda_{\rm c}\int_0^\infty\Big(1-\sum_{k=0}^{M}\rho(\nu)^k \times \frac{\bar{m}^k e^{-\bar{m}}}{k!\eta}\Big)\nu{\rm d}\nu\Big),$$

with $\rho(\nu)=\int_0^\infty \frac{1}{1+su^{-\alpha}} f_U(u|\nu){\rm d}u$ where (a) follows from expectation over $h_{y_x}\sim\exp(1)$, (b) follows from the expectation over number of interfering devices per cluster, (c) follows from the probability generating functional (PGFL) of PPP [13] along with converting from Cartesian to polar coordinates. Now, under the assumption of $\bar{m} \ll M$, the Laplace transform of inter-cluster interference reduces to (16).

REFERENCES

[1] K. Doppler, M. Rinne, C. Wijting, C. B. Ribeiro, and K. Hugl, "Device-to-device communication as an underlay to LTE-advanced networks," *IEEE Commun. Magazine*, vol. 47, no. 12, pp. 42–49, Dec. 2009.

[2] M. Cha, H. Kwak, P. Rodriguez, Y.-Y. Ahn, and S. Moon, "I Tube, You Tube, Everybody Tubes: analyzing the world's largest user generated content video system," in *Proc., ACM Intl. Conf. on Special Interest Group on Data Commun. (SIGCOMM)*, 2007.

[3] S. Krishnan and H. S. Dhillon, "Distributed caching in device-to-device networks: A stochastic geometry perspective," to appear in *Proc. Asilomar*, Pacific Grove, CA, Nov. 2015.

[4] N. Golrezaei, P. Mansourifard, A. Molisch, and A. Dimakis, "Base-station assisted device-to-device communications for high-throughput wireless video networks," *IEEE Trans. on Wireless Commun.*, vol. 13, no. 7, pp. 3665–3676, Jul. 2014.

[5] M. Ji, G. Caire, and A. F. Molisch, "Wireless device-to-device caching networks: Basic principles and system performance," submitted to *IEEE Journal on Sel. Areas in Commun.*, 2014, available online: arxiv.org/abs/1305.5216.

[6] ——, "Fundamental limits of caching in wireless D2D networks," submitted to *IEEE Trans. on Info. Theory*, 2014, available online: arxiv.org/abs/1405.5336.

[7] H. ElSawy and E. Hossain, "Analytical modeling of mode selection and power control for underlay D2D communication in cellular networks," *IEEE Trans. on Commun.*, vol. 62, no. 11, pp. 4147–4161, Nov. 2014.

[8] G. George, R. K. Mungara, and A. Lozano, "An analytical framework for device-to-device communication in cellular networks," 2014, available online: arxiv.org/abs/1407.2201.

[9] A. H. Sakr and E. Hossain, "Cognitive and energy harvesting-based D2D communication in cellular networks: Stochastic geometry modeling and analysis," *IEEE Trans. on Commun.*, vol. 63, no. 5, pp. 1867–1880, May. 2015.

[10] X. Lin, R. Ratasuk, A. Ghosh, and J. G. Andrews, "Modeling, analysis and optimization of multicast device-to-device transmissions," *IEEE Trans. on Wireless Commun.*, vol. 13, no. 8, pp. 4346–4359, Aug. 2014.

[11] M. Afshang, H. S. Dhillon, and P. H. J. Chong, "Coverage and area spectral efficiency of clustered device-to-device networks," in *Proc. IEEE Globecom*, San Diego, CA, Dec. 2015.

[12] R. K. Ganti and M. Haenggi, "Interference and outage in clustered wireless ad hoc networks," *IEEE Trans. on Info. Theory*, vol. 55, no. 9, pp. 4067–4086, Sep. 2009.

[13] D. Stoyan, W. S. Kendall, and J. Mecke, *Stochastic Geometry and its Applications*, 2nd ed. New York: John Wiley and Sons, 1995.

[14] M. Afshang, H. S. Dhillon, and P. H. J. Chong, "Modeling and performance analysis of clustered device-to-device networks," submitted to *IEEE Trans. on Wireless Commun.*, available online: arxiv.org/abs/1508.02668.

[15] H. A. David and H. N. Nagaraja, *Order Statistics*. New York: John Wiley and Sons, 1970.